RD Instruments

RD Instruments Technical Note:

Why An ADCP Doesn't Need Compass Data for Discharge Calculation

Summary: *This note presents a mathematical proof that discharge calculated from the ADCP measured velocities in earth coordinates is the same as that calculated from the velocities in ADCP instrument coordinates. That is, velocities in ADCP coordinates do not need to be converted into velocities in earth coordinates when calculating discharge. Therefore, compass data, which are used to convert velocities from ADCP coordinates to earth coordinates, are not needed for discharge calculation. In other words, an ADCP can measure discharge without being equipped with a compass, provided that bottom tracking by the ADCP is valid and used for deriving boat velocity.*

An ADCP measures water velocity by the so-called water tracking and boat velocity by the so-called bottom tracking methods. These velocities are measured relative to the ADCP. The absolute water velocity is derived by subtracting boat velocity from the water tracking velocity, assuming no moving bottom exists. Both water velocity and boat velocity are collected using the same coordinate system: ADCP beam coordinates are converted using a simple transformation into ADCP instrument coordinates X-Y-Z. The velocities in X-Y-Z coordinates can be converted to earth coordinates (East-North-Up) using compass data. However, for discharge calculation, the conversion from X-Y-Z coordinates to earth coordinates is not needed and discharge can be calculated directly using velocities in X-Y-Z coordinates. Because of this, compass data are not needed for discharge calculation.

The general equation for determining river discharge is written as follows:

$$Q = \iint_S u \bullet \xi \, ds \qquad (1)$$

where Q is the discharge, S is the cross-section area along a boat's track, u is the absolute water velocity vector in earth-coordinates, and ξ is a unit vector normal to the boat's track at a differential area ds. ds is determined by the following:

$$ds = |V_b| \cdot dz \cdot dt \qquad (2)$$

where dz is the differential depth, dt is the differential time, and V_b is the boat velocity vector and $|V_b|$ is the boat speed, z is the vertical coordinate, i.e. z=0 is the river bottom, and z=H is the water surface (H being the water depth along the boat's track). $|V_b|$ is determined from the following:

$$|V_b| = \sqrt{V_{bE}^{\,2} + V_{bN}^{\,2}} \qquad (3)$$

where V_{bE} and V_{bN} are the east and north components of the boat velocity vector, respectively.

Eq. (1) can be re-written as:

$$Q = \int_0^T \left[\int_0^H u \cdot dz \right] \bullet \xi \, | V_b | \cdot dt = \int_0^T \int_0^H (u \times V_b) \bullet k \cdot dz \; dt \qquad (4)$$

where T is the total transect time, and k is the unit vector in the vertical direction.

Let f denote the cross-product in Eq. 4:

$$f = (u \times V_b) \bullet k = u_E V_{bN} - u_N V_{bE} \qquad (5)$$

where subscripts E and N stand for components in the east and north direction, respectively.

Our goal is to prove that the cross-product f calculated in earth coordinates will be the same as that calculated in ADCP instrument coordinates. That is, f is independent from coordinate system for velocities.

Assume the Z direction in ADCP instrument coordinates is the same as the Up direction in earth coordinates and there is an angle _ (i.e., heading measured by a compass) between the X (or Y) direction and East (or North) direction. Based on coordinate conversion formula that can be found in mathematical text books, the horizontal velocities in earth coordinates can be written as a function of the velocities in X-Y-Z coordinates and heading _:

$$u_E = u_x \cos\theta - u_y \sin\theta \qquad \textbf{(6)}$$

$$u_N = u_x \sin\theta + u_y \cos\theta \qquad \textbf{(7)}$$

$$V_{bE} = V_{bx} \cos\theta - V_{by} \sin\theta \qquad \textbf{(8)}$$

$$V_{bN} = V_{bx} \sin\theta + V_{by} \cos\theta \qquad \textbf{(9)}$$

Substitute Eqs. 6 through 9 into Eq. 5:

$$\begin{aligned} f &= u_E V_{bN} - u_N V_{bE} \\ &= (u_x \cos\theta - u_y \sin\theta) \bullet (V_{bx} \sin\theta + V_{by} \cos\theta) - (u_x \sin\theta + u_y \cos\theta) \bullet (V_{bx} \cos\theta - V_{by} \sin\theta) \end{aligned} \qquad \textbf{(10)}$$

Write out all of the terms in the right hand of Eq. 10. The terms with $sin\_\_cos\_$ are all cancelled out. Only terms with $sin^2\_$ or $cos^2\_$ are left:

$$f = u_E V_{bN} - u_N V_{bE} = u_x V_{by} (\sin^2 \theta + \cos^2 \theta) - u_y V_{bx} (\sin^2 \theta + \cos^2 \theta) \qquad (11)$$

Note that $sin^2\_ + cos^2\_ = 1$. Then, Eq. 11 becomes:

$$f = u_E V_{bN} - u_N V_{bE} = u_x V_{by} - u_y V_{bx} \qquad \textbf{(12)}$$

Eq. 12 indicates that the cross-product f calculated in earth coordinates is the same as that calculated in ADCP instrument coordinates. That is, the cross-product is coordinate independent. Note that heading _ disappears from Eq. 12. This means that compass data (i.e., heading _) for coordinate conversion are not needed for discharge calculation.

It is important to note that the above argument is valid only when the boat velocity is derived from ADCP bottom tracking data. If DGPS data are used to derive boat velocity, compass data are required for discharge calculation.

References

Simpson, M. R., and Oltmann, R. N. (1993). "Discharge-Measurement system using an acoustic Doppler current profiler with application to large rivers and estuaries." United States Geological Survey, Water-Supply Paper 2395.